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# Removal of Colloidal Particles in Electroflotation

**The collection mechanism of polystyrene latices of 0.6 μm in diameter in electroflotation has been examined both theoretically and experimentally as extension of our previous work (1980). To avoid the change of surface state of particle and bubble, the experiment was conducted under the condition that the concentration of CTAB (cetyl trimethylammonium bromide), cationic surfactant in the solution, is within the 90% of its initial concentration. It is of interest that the theory predicts the collection efficiency to vary as $d_i^{-1.91}$.**

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## SCOPE

**The removal of suspended substances from effluents is important in waste water treatment from the environmental viewpoint. The particles to be removed in effluent treatment are normally less than about 20 μm and close to neutral buoyancy, and the concentration is also very dilute as low as 20 ppm.**

**Since it has been found effective empirically, to use very small bubbles often less than 100 μm in dia. for the removal of fine particles, electroflotation has become of interest. The diameter of bubbles generated by electrolysis is around 20 μm. This method is also advantageous, compared with the usual treatment technique in many aspects.**

1) **The apparatus is small and compact.**
2) **The system can be controlled electrically corresponding to the amount of floc.**
3) **The temperature of waste water does not affect the generation of gas bubbles even when it is high.**

**The electroflotation has been first applied to the treatment of domestic sewage in 1911 in the U.S. (Siegerman, 1971). At that time, however, because the electrodes tended to scum and scale stuck on the surfaces of the electrodes, the device prevented electric current from flowing. In Japan, several companies recently have solved these problems by developing their system in actual effluent treatment.**

**Though the actual system has been developed as noted above, the theoretical study of electroflotation itself has not been widely developed. The objective of this paper is to get information both theoretically and experimentally on collection mechanism which is of importance for the planning of actual apparatus.**

**In the recent study of flotation, Flint and Howarth (1971) calculated the collision efficiencies relating to a 6 μm galena particle and air bubble of diameter between 50 and 100 μm from limiting trajectories. Reay and Ratcliff (1975) have reported that the rate of flotation of glass beads varied approximately as the 1.5 power of particle diameter. They have suggested that the most fruitful course for future work would appear to be a series of experiments aimed at discovering how the flotation rate vs. particle size relationship depends on the zeta potential of the particles. Collins and Jameson (1976) have revealed that the rate**

**of flotation of polystyrene particles has the similar dependence on the particle size (Reay and Ratcliff, 1975). Collins and Jameson (1977) have also pointed out that the rate of flotation deeply depends on the charge of both particle and bubble while they (1978) have developed a new technique to measure small gas bubbles. Our previous work on electroflotation of submicron particles (1980) has presented the collection efficiency by solving the diffusion equation, considering the double layer repulsive and London attractive forces.**

**In this work, we extend our previous work to include the effects of increase of particle drag due to the fluid between particle and bubble and the size distribution of bubbles. The experimental and calculated results are discussed and compared in detail.**

## CONCLUSION AND SIGNIFICANCE

**The process of electroflotation for submicron particles has been quantitatively evaluated by calculating collection efficiency. London attractive and double layer repulsive forces, the hydrodynamic interaction and the size distribution of bubbles have been taken into consideration in our calculation. To keep surface properties unchangeable, the experiment was conducted as the concentration of cationic surfactant CTAB maintained 90% of initial concentration during a run. Therefore, the Hamaker constant and the zeta potentials of particle and bubble are considered to be constant during a run. The size distribution of bubbles is also included in our calculation.**

**When the average diameter of bubbles is used for calculation, the number of bubbles is overestimated; as a result, the total collection efficiency is also overestimated. The theory predicts that the collection efficiency drastically decreases even for the small value of products of zeta potentials when the effective Hamaker constant is below about 3 × $10^{-21}$J, whereas it keeps nearly constant even for the value of product of zeta potentials such as about 1,400 (mV)$^2$ when Hamaker constant is above about 5 × $10^{-21}$J. This means that the efficiency deeply depends on both the effective Hamaker constant and the value of product of zeta potentials. The Hamaker constant was chosen 1 × $10^{-20}$J in our calculation to explain experimental results well. Therefore, the difference of efficiency caused by the effect of zeta potentials was very small in our range of experiment.**

**The calculated results indicate that the collection efficiency is nearly proportional to $d_i^{-1.91}$. Since the number of particles which are actually collected by a bubble per unit time is expressed as $\eta_\epsilon\, \pi/4\, d_i^2 V_{0i}$, this number of particles is proportional to the rising velocity of a bubble. This means that the number of particles which are actually collected by a bubble per unit time increases with the increase of diameter of bubble as far as our experimental range is concerned. The theory predicts that, to increase the rate of flotation in the actual system of electroflotation, zeta potentials of bubble and particle should be kept opposite in sign, and the "hydrophobicity" of the surfaces of both particle and bubble should be increased by surfactant if possible.**

### PREVIOUS WORK

Flint and Howarth (1971) calculated trajectories for particles in the path of a spherical bubble rising in an infinite pool of liquid to evaluate collection efficiencies from limiting trajectories. They reported that collision efficiencies of fine particles (6 μm galena particles) are shown to be independent of the chosen flow field around bubble. Reay and Ratcliff (1973, 1975) predicted a minimum in the collection efficiency at a particle size around 1 μm in diameter by theoretical approach based on concepts developed in the field of aerosol filtration. They also reported that the flotation rate of glass beads was proportional to 1.5 power of particle diameter, but for the polystyrene particles the exponent was about 0.5. Collins and Jameson (1976) later supported the choice of 1.5 power dependence by their experiment with polystyrene. Reay and Ratcliff (1975) suggested that the most fruitful course for future work would appear to be a series of experiments aimed at discovering how the flotation rate depends on the zeta potential of the particles, coupled with the development of a technique for measuring the zeta potential of bubbles. Collins and Jameson (1977) carried out experiments on polystyrene latices of diameter between 4 and 20 μm to examine how the zeta potential of particle and bubble influence the rate of flotation. They concluded that the rate of flotation drastically depends on the charge on both the bubble and the particle and have proposed a simple correlation for the effect of particle and bubble charges on the flotation rate constant such as:

$$-ln(k_p/d_p^{1.5}) = 3.9 + 0.116 u_E u_B \qquad (1)$$

They also succeeded in measuring the charge on small gas bubbles by their new technique (1978).

Our previous work (1980) on electroflotation of submicron particles presented the collection efficiency by solving the diffusion equation around a bubble including the surface interaction of bubble and particle such as London attractive and double layer repulsive forces. The influence of zeta potentials of bubble and particle on the rate of flotation has been explained when the Hamaker constant was arbitrarily chosen as 3 × $10^{-21}$J. The method to measure the charge on gas bubbles followed a technique developed by Collins et al. (1978). The range of our measurement could be widen by using video technique (1982).

### THEORY

The theory is substantially the same as the result of our earlier paper (1980). The main change is to take into account bubble size distribution and the effect of hydrodynamic drag increase which was not exactly considered in the earlier paper, because only the first three terms in infinite series derived by Wakiya (1957) for that phenomena was considered.

A correction factor $\beta$ for Stokes' expression necessitated by the presence of the plane boundary at a infinite distance from the particle was also obtained by Brenner (1961). Our experimental work on this effect (1981) is one of the verifications of his theoretical study. In this paper, the following approximate equation for $\beta$ of Brenner proposed by Honig et al. (1971) is used because of mathematical simplicity and handiness.

$$\beta = \frac{6\bar{h}^2 + 13\bar{h} + 2}{6\bar{h}^2 + 4\bar{h}} \qquad \bar{h} = \frac{h}{a_p} \qquad (2)$$

Under suitable assumptions, the diffusion equation is expressed in two dimensional form as:

$$\begin{aligned}\bar{v}_r \frac{\partial \bar{C}}{\partial \bar{r}} + \bar{v}_0 \frac{1}{\bar{r}} \frac{\partial \bar{C}}{\partial \theta} = \frac{\bar{D}}{\beta} \Bigg[ & \left\{ \frac{2}{\bar{r}} + \frac{d\bar{\phi}}{d\bar{r}} + \beta \frac{d}{d\bar{r}}\left(\frac{1}{\beta}\right) \right\} \frac{\partial \bar{C}}{\partial \bar{r}} \\ & + \left\{ \frac{1}{\bar{r}} \frac{d\bar{\phi}}{d\bar{r}} + \frac{1}{\bar{r}^2 \tan\theta} \right. \\ & \left. + \frac{\beta}{\bar{r}} \frac{d}{d\bar{r}}\left(\frac{1}{\beta}\right) \right\} \frac{\partial \bar{C}}{\partial \theta} + \frac{\partial^2 \bar{C}}{\partial \bar{r}^2} \\ & + \left\{ \frac{2}{\bar{r}} \frac{d\bar{\phi}}{d\bar{r}} + \frac{d^2\bar{\phi}}{d\bar{r}^2} + \beta \frac{d}{d\bar{r}}\left(\frac{1}{\beta}\right) \frac{d\bar{\phi}}{d\bar{r}} \right\} \bar{C} \Bigg] \end{aligned} \qquad (3)$$

The boundary conditions for Eq. 3 are as:

$$\overline{C} = 1.0 \text{ at } \bar{r} = \infty$$
$$\overline{C} = 0.0 \text{ at } \bar{r} = 1 + \frac{a_p}{a} \qquad (4)$$

To solve the concentrationa profile around the collector by the Crank-Nicolson method, the concentration on the axis ($\theta = 0$) need to be known as one of boundary conditions. However, it is difficult to compute the diffusion equation on the axis ($\theta = 0$) by using the Runge-Kutta method in the region where $h$ is nearly equal to zero unlike the case in the earlier paper. When the correction factor of $\beta$ was excluded, the numerical computation was carried out without much difficulty. Therefore, the concentration distribution on the axis has been calculated in the following way.

The dimensionless local flux along the axis is given as:

$$\overline{N} = -\frac{\overline{D}}{\beta}\left(\frac{d\overline{C}}{d\bar{r}} + \overline{C}\frac{d\bar{\phi}}{d\bar{r}}\right) \qquad (5)$$

The solution of this equation for the concentration of particles at $\bar{r}$ under the boundary conditions described above is:

$$\overline{C} = \exp(-\bar{\phi}) \int_{\bar{r}_0}^{\bar{r}} \exp(\bar{\phi})\beta\left(\frac{-\overline{N}}{\overline{D}}\right) d\bar{r} \qquad (6)$$

Since $\overline{N}$ is dependent on $\bar{r}$, $\overline{N}$ is expressed as:

$$\overline{N} = \overline{N}_0 f(\bar{r}) \qquad (7)$$

where $N_0$ is the dimensionless flux at $\bar{r} = \bar{r}_0$ and $f(r)$ is assumed as follows from the law of conservation of mass.

$$f(\bar{r}) = \begin{cases} \left(\dfrac{\bar{r}_0}{\bar{r}}\right)^2 & \text{for } \bar{r}_0 \leq \bar{r} \leq \bar{r}_1 \\ \left(\dfrac{\bar{r}_0}{\bar{r}_1}\right)^2 \left(\dfrac{\bar{r}_\infty - \bar{r}}{\bar{r}_\infty - \bar{r}_1}\right) & \text{for } \bar{r}_1 < \bar{r} \end{cases} \qquad (8)$$

Here $\bar{r}_\infty$ denotes $\bar{r} = \infty$ where $\overline{C} = 1.0$; $\bar{r}_1$ denotes $\bar{r}$ where the effect of double layer repulsion potential energy disappears and the potential curves arrange themselves on a single curve. Equation 8 means that the particles on the axis ($\theta = 0$) are assumed to move along the axis toward the center of collector and not to drift to the $\theta$ direction. Then the concentration distribution on the axis is given as:

$$\overline{C} = -\frac{\overline{N}_0}{\overline{D}} \exp(-\bar{\phi}) \int_{\bar{r}_0}^{\bar{r}} \exp(\bar{\phi})\beta f(\bar{r}) d\bar{r} \qquad (9)$$

which can be computed numerically by Simpsons's method if $\overline{N}_0$ is known. After the concentration is calculated for an arbitrarily given $\overline{N}_0$ because the value of $\overline{N}_0$ is unknown, the concentration is determined by multiplying a factor so that $\overline{C}$ at $\bar{r} = \infty$ becomes 1.0. Since the concentration profile on the axis ($\theta = 0$) can be known in this way, Eq. 3 can be numerically solved by the Crank-Nicolson method to yield the concentration distribution of the total region around the collector.

Accordingly, after calculating the collection efficiency, the total collection efficiency is derived from a mass balance considering the size distribution of bubbles generated for $dt$ as:

$$E = 1 - \frac{C}{C_0} = 1 - \exp\left(-\frac{3R_GTI}{8PF_aA_s} \cdot \frac{K_3}{K_1K_2} t\right) \qquad (10)$$

where

$$K_1 = \sum_i a_i^3 P(a_i) \qquad K_2 = \sum_i V_{0i} P(a_i) \qquad K_3 = \sum_i \eta_{ti} a_i^2 V_{0i} P(a_i) \qquad (11)$$

The rise velocities of bubbles are calculated by the following equation.

$$V_{0i} = \frac{3}{2} \cdot \frac{(\rho - \rho_B)}{18\mu} g d_i^2 \qquad (12)$$

The precise information for calculation is referred to the previous paper.

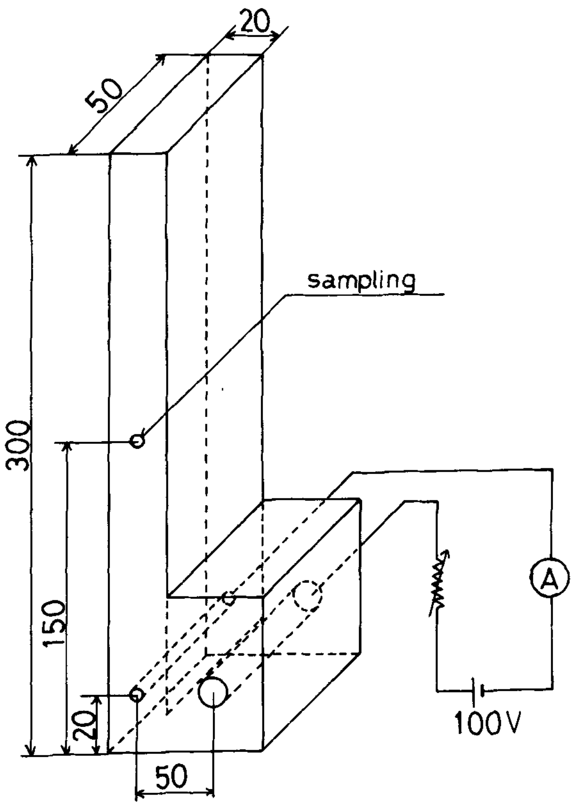

**Figure 1. Schematic diagram of experimental setup.**

## EXPERIMENT

The flotation experiments were carried out batchwise in an electrolysis vessel, Figure 1. The aqueous solution under test contained the cationic surfactant to promote flotation. The surfactant used was cetyl trimethylammonium bromide (CTAB) and the initial concentration of CTAB was always the same, $5 \times 10^{-2}$ mol/m$^3$. The zeta potentials of both particle and bubble were controlled by addition of $Na_2SO_4$ in the range 0.5 to 10 mol/m$^3$. Addition of the $Na_2SO_4$ in this range caused the charges to decrease. The particles used were of polystyrene latices of 0.6 $\mu$m in diameter with standard deviation 0.0076 $\mu$m. Before a run, the solution was mixed well to let the particles disperse and was kept still for half an hour. After gently pouring the solution into the electrolysis vessel, the electric current was made flow by controlling with a rheostat. Since CTAB is adsorbed by bubbles during a run, the surface property would change with flotation time. To avoid this, the experiment was carried out within the time when the amount of CTAB remained 90% of the initial amount (Appendix). Accordingly, the concentration of CTAB was regarded as nearly constant and so the surface property was regarded as unaltered during a run. The solution was sampled with a syringe from a small hole of the vessel before and during a run, and its particle concentration was examined with a spectrophotometer (JASCO UV-50). After examining the absorbance of

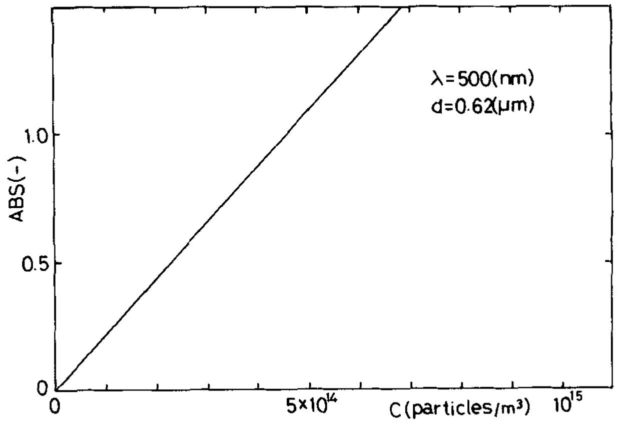

**Figure 2. Relation of absorbance and particle concentration. The wave length used for measurement was 500 nm.**

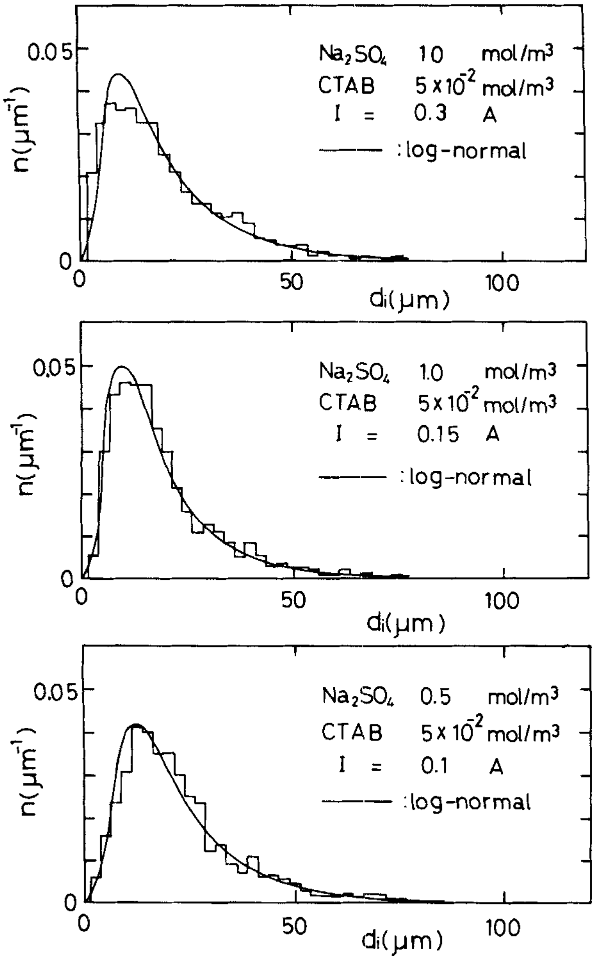

**Figure 3. Distribution of bubble sizes for different electric currents and concentrations of $Na_2SO_4$. The solid line is log-normal distribution curve (Eq. 21).**

solution, the number concentration of particles in the small cell was counted by using a microscope, T.V. camera, and monitor T.V. Since the volume of the screen was known from the depth of focus and magnification of the micrometer, the number of particles divided by the volume gave the number concentration of particles. The relation of absorbance and particle concentration is shown in Figure 2. The size distribution of bubbles, introduced into oil spread on the top surface of the solution, was examined by using a microscope, T.V. camera, and monitor T.V.

The zeta potential of particles was examined by using a Mitamura microelectrophoresis cell and the measurement of zeta potential of bubbles followed the method by Collins et al. (1980). The results of zeta potential of bubble are presented in our previous paper (1982). The coagulation of particles was checked for each sample of solution with a microscope but was not observed during a run in our experiment.

## RESULTS AND DISCUSSION

The size distribution of bubbles for each concentration of $Na_2SO_4$ are shown in Figure 3. The distribution curve is well expressed by the log-normal distribution as:

$$P(d_i) = \frac{1}{\sqrt{2\pi} d_i ln\sigma} \exp\left[-\frac{(lnd_n - lnd_i)}{2(ln\sigma)^2}\right] \qquad (13)$$

The zeta potentials of the particle and bubble are given in Figure 4, along with the data of Collins et al. (1980). The values of zeta potential of particles are rather small compared with the results of Collins et al., while those of bubbles are almost the same as their results. The difference of results of particle charge might have been due to the difference in polymerizing or cleaning method. However, the tendency that the charges on both particle and bubble decrease with increasing concentration of $Na_2SO_4$ is clearly shown in both cases. Our measurement has been succeeded in widening the experimental range.

The time $T_c$ during which the concentration of CTAB keeps more than 90% of initial one so that the surface property is regarded as constant and stable is calculated for various concentration of

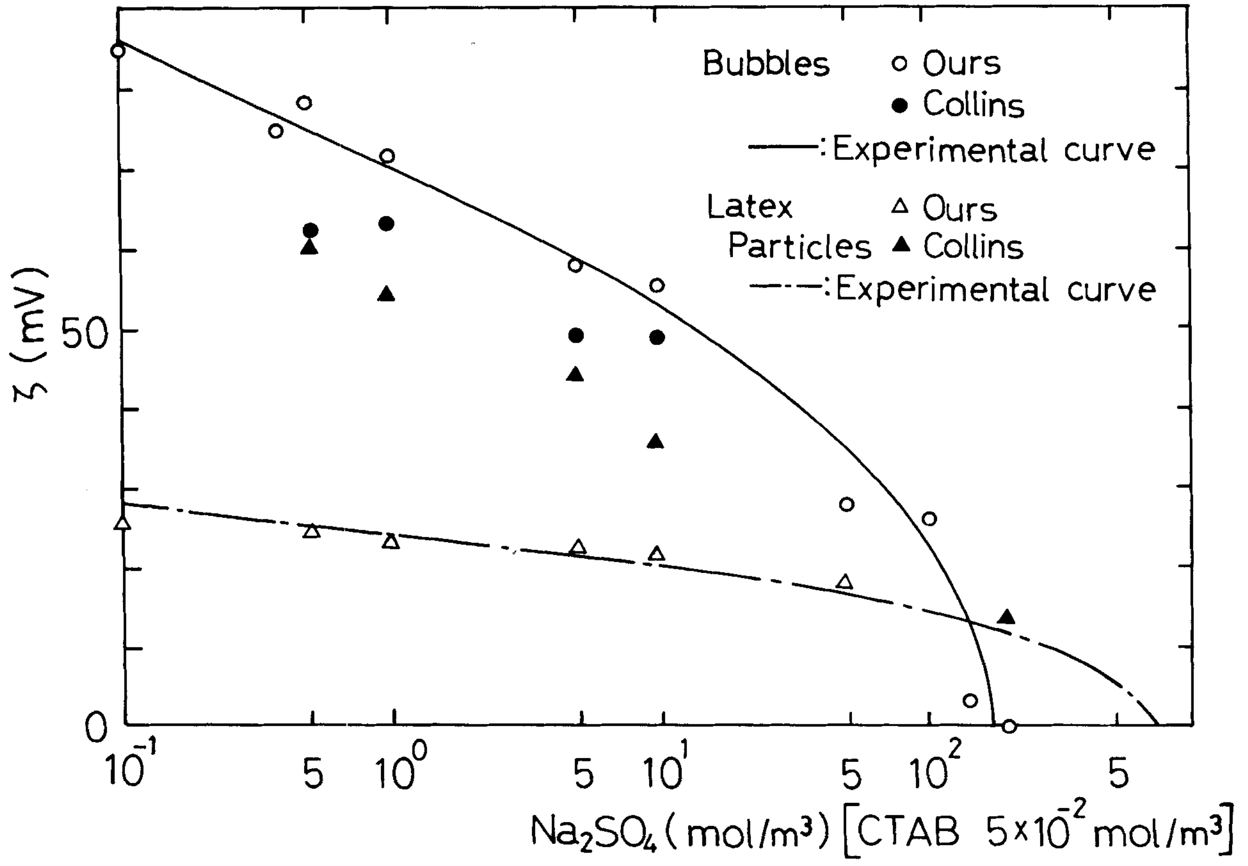

**Figure 4. Zeta potentials of particle and bubble in the solution of $Na_2SO_4$ and CTAB along with the results of Collins et al. (1978).**

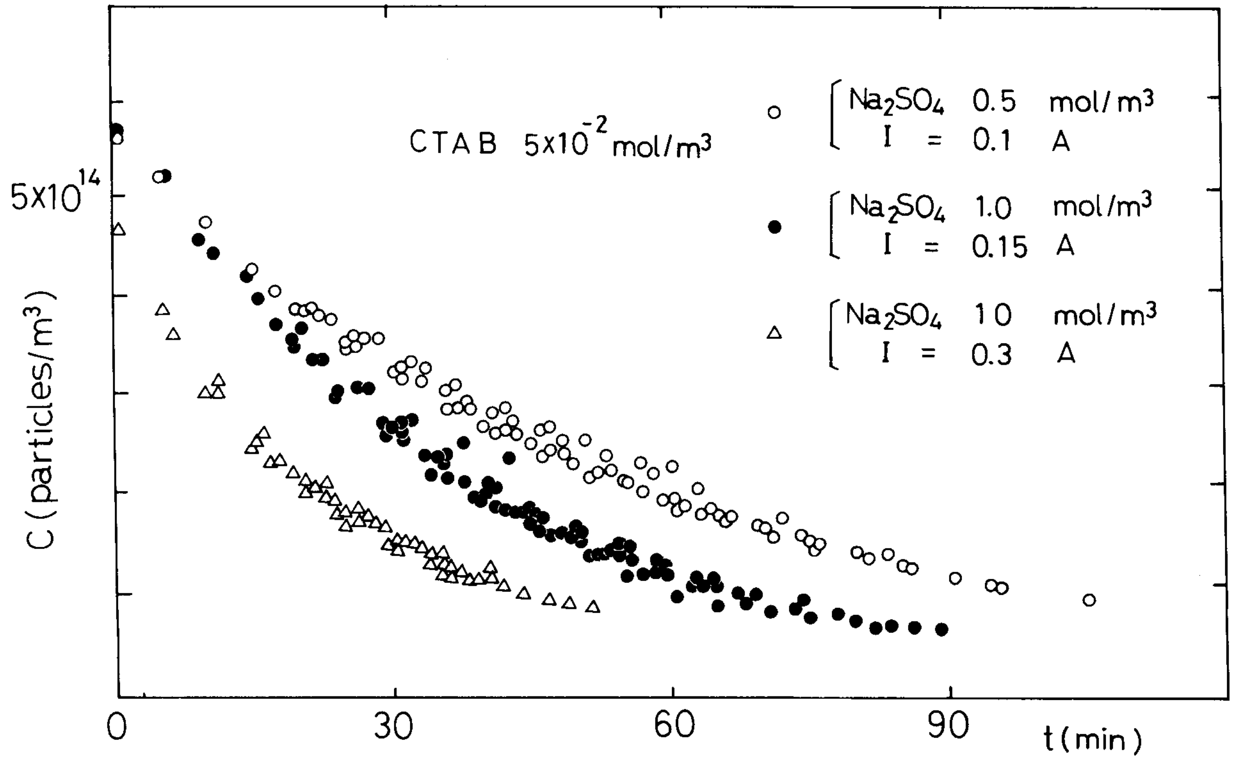

**Figure 5. Change of particle concentration with time. The data within $T_c$ are replotted so that the concentration of CTAB is considered to be constant.**

$Na_2SO_4$ as follows. $T_c$'s are 40, 25 and 15 min when the concentrations of $Na_2SO_4$ are 0.5, 1 and 10 mol/m$^3$, respectively. $T_c$ decreases with increasing concentration of $Na_2SO_4$ because the bubble concentration increases with increasing electric current. Therefore, the flotation experiment was conducted by changing initial concentration of particle so that the result of every run within each $T_c$ was gathered and arranged for change of particle concentration as in Figure 5.

The result of theoretical single collection efficiencies is shown for various size of bubbles when the concentration of $Na_2SO_4$ is 0.5 mol/m$^3$ in Figure 6. When the concentration of $Na_2SO_4$ is 1 and 10 mol/m$^3$, the result of theoretical single collection efficiencies are almost the same as that in Figure 6. This is because London attractive force is so strong that the increase in the product of zeta potentials of particle and bubble as in our experiment does not affect the noticeable decrease in collection efficiency. The value of Hamaker constant is regarded as $1 \times 10^{-20}$J in our calculation.

A few workers have investigated the effect of bubble size on the rate of flotation. The bubble size is the parameter that is most difficult to vary in flotation, and this is the probable reason for the relative paucity of experimental data on its effects.

The experimental data by Reay and Ratcliff (1975) supported that collection efficiency should vary as $d_i^{-2}$ for smaller bubbles than 100 $\mu$m. The particles used were glass beads and polystyrene latices. The electrical interactions were assumed to be neglected.

Anfruns and Kitchner (1977) studied the rate of capture of small particles (quartz or glass beads), generating single bubbles. The collection efficiency from their experimental data was proprotional to $d_i^{-1.67}$. Jameson et al. (1977) summarized these results as:

$d_p$ 4 to 30 $\mu$m; $d_i < 100$ $\mu$m

$$\eta \alpha d_i^{-2} \tag{14}$$

$d_p$ 10 to 50 $\mu$m; $d_i$ 600 to 1,000 $\mu$m

$$\eta \alpha d_i^{-1.67} \tag{15}$$

Our calculated result, considering electrical interaction, indicates:

$d_p = 0.6$ $\mu$m; $d_i < 75$ $\mu$m

$$\eta \alpha d_i^{-1.91} \tag{16}$$

The value of power in our result is very similar to that derived by Reay and Ratcliff (1975). Our result supports the empirical ob-

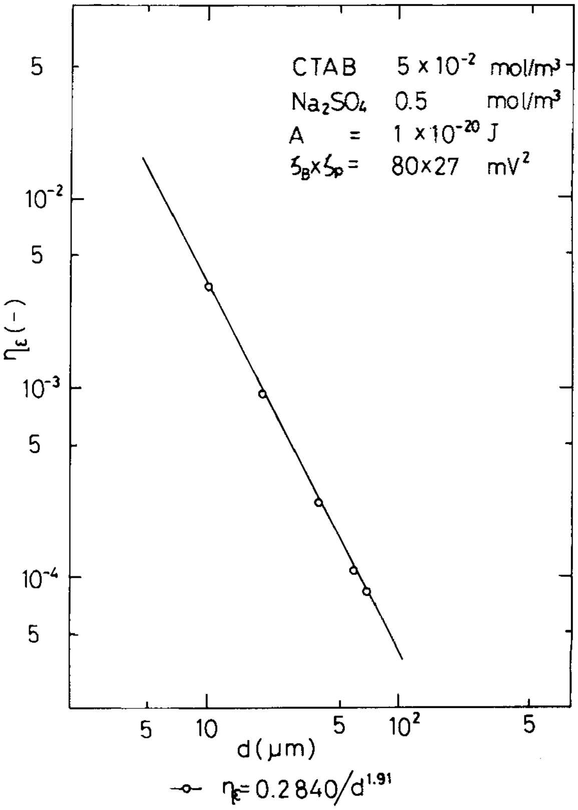

**Figure 6. Relation of collection efficiency and bubble size. White circles are the calculated results and the solid line is expressed by the equation in the figure.**

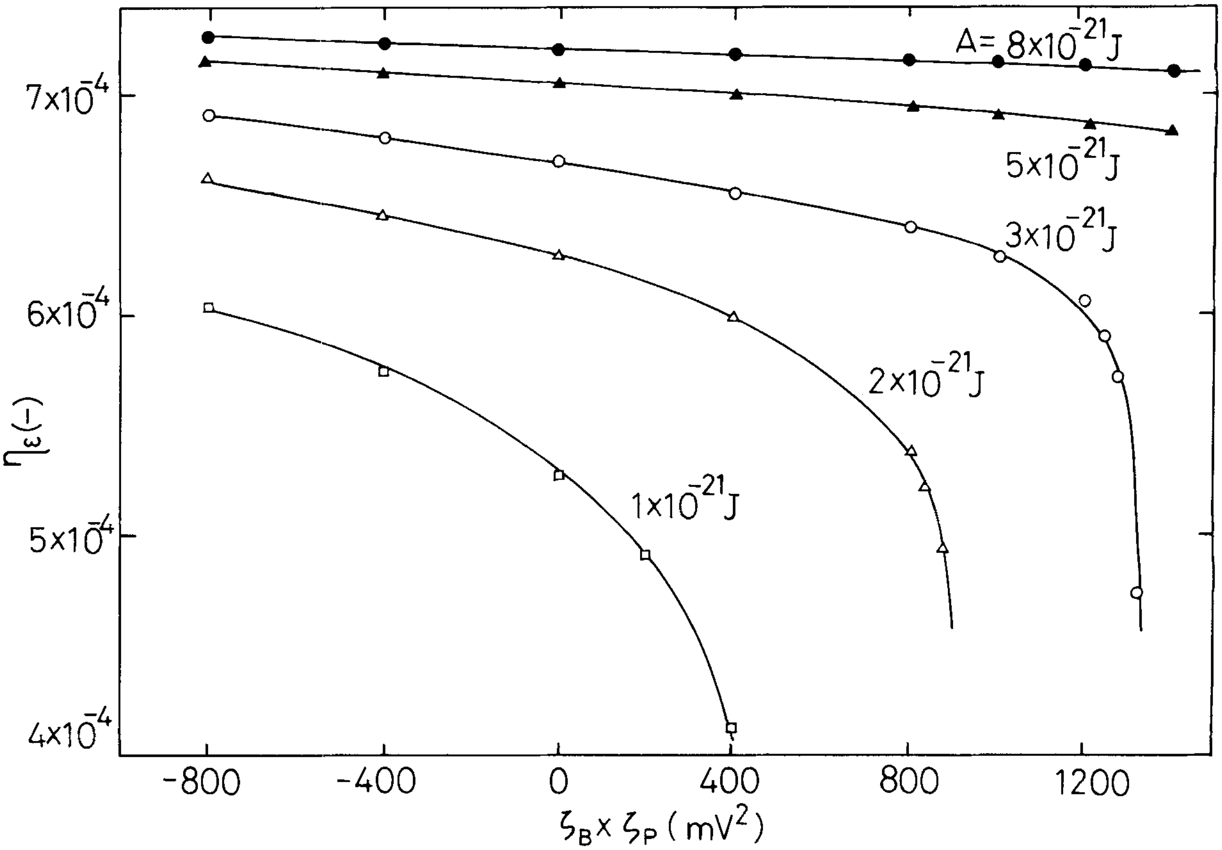

**Figure 7. Collection efficiencies of bubbles of diameter 20 $\mu$m for 0.6 $\mu$m particle. The Debye length was fixed as 1.27 nm for simplicity in calculation which is identical with the Debye length when the 1-1 ion concentration is 1.5 mol/m$^3$. The symbols are the calculated results.**

servation that bubbles less than 100 $\mu$m are effective for fine particle flotation as far as our experimental range is concerned in which the flow around a bubble is in Stokes region and Hamaker constant, and the product of zeta potentials are those as noted above.

It is of interest to know the dependence of efficiency on Hamaker constant and the products of zeta potentials. Accordingly, the collection efficiencies under various conditions are summarized in Figure 7, although the trends predicted by the theory have never been subjected to systematic experimental investigation and predictions cannot be accepted with confidence. When Hamaker constant is smaller than about $3 \times 10^{-21}$J, the efficiency drastically

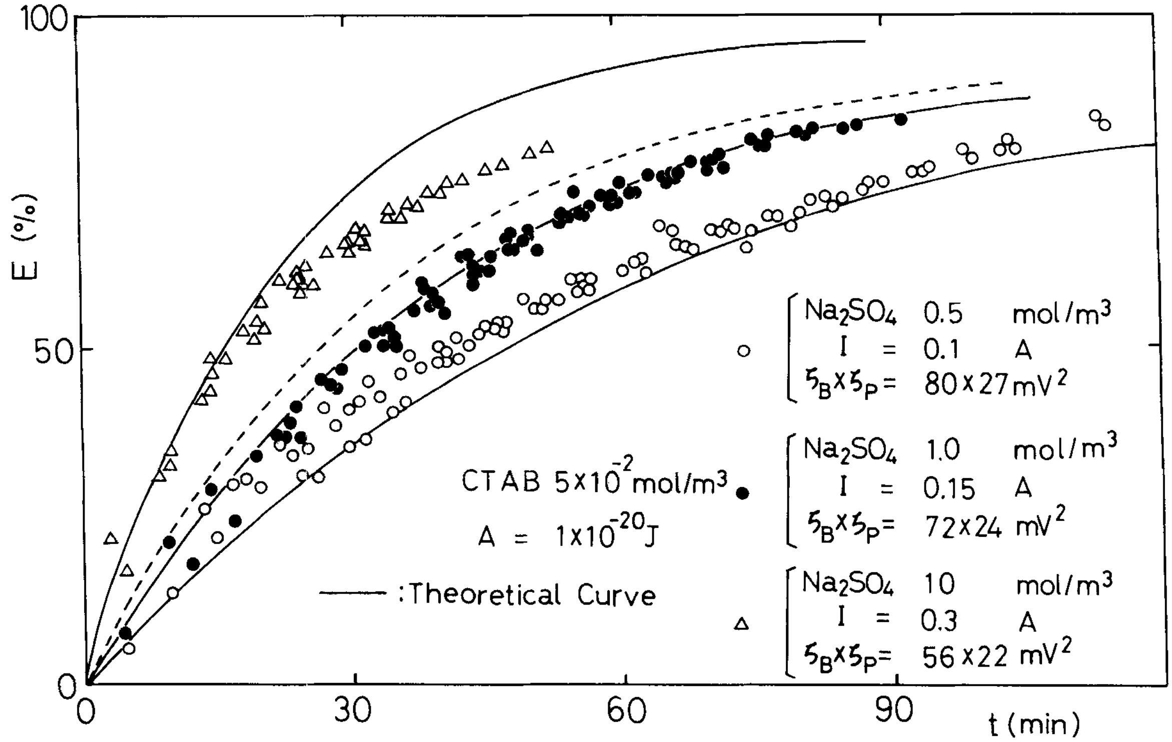

**Figure 8. Comparison of theoretical and experimental total collection efficiencies. The dotted line is the theoretical curve calculated by using the average diameter of bubbles when the concentration of Na$_2$SO$_4$ is 0.5 mol/m$^3$.**

decreases when the product of zeta potentials becomes over a certain value. On the other hand, when Hamaker constant is larger than about $5 \times 10^{-21}$J, the efficiency keeps nearly constant in this range of product of zeta potentials. This supports the results of single collection efficiencies when Hamaker constant is $1 \times 10^{-21}$J as discussed above. Undoubtedly, the efficiency increases with increase of the value of Hamaker constant for the same value of product of zeta potentials.

Theoretical collection efficiency which is calculated by using Eq. 10 is compared with the experimental one in Figure 8. The theoretical collection efficiencies for lower electric currents are in close agreement with the experimental results. However, when the electric current becomes 0.3 A, the discrepancy of the predicted curve from the experimental result is observed. This is due to the effect of convection which was observed in that case. Since the Hamaker constant was arbitrarily chosen for calculation, the quantitative difference of theoretical and experimental values is not so significant. The qualitative similarity of shape of the theoretical curve and experimental results is rather more important. [The Hamaker constant in case of coagulation of latex particles was determined to be about $8 \times 10^{-21}$J by Higashitani et al. (1978) which is in fair agreement with the result by Lyklema (1968).] When the average diameter of bubbles on number basis was used for theoretical calculation, neglecting the size distribution of bubbles, the predicted result overestimates the experimental result as shown by dotted line in Figure 8. This is because the number of bubbles is overestimated by using the average diameter of bubbles on number basis. (The number concentration of bubbles estimated by average diameter on number basis is about three times as high as that estimated by using size distribution when the electric current is 0.1 A.) If the number of bubbles is correctly estimated, the average diameter on area basis can be used for the correct estimation of total collection efficiency.

## ACKNOWLEDGMENT

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## NOTATION

$A$ = Hamaker constant
$A_s$ = cross-sectional area of vessel
$a$ = radius of bubble
$a_p$ = radius of particle
$C_0$ = concentration of particles far from bubble
$C$ = local concentration of particles
$C'$ = bulk concentration of surfactant
$D$ = diffusion coefficient
$d_i$ = diameter of bubble $i$
$d_p$ = diameter of particle
$d_m$ = most probable diameter of bubble
$E$ = total collection efficiency
$Fa$ = Faraday constant
$h$ = minimum separation between bubble and particle surface
$i$ = bubble of diameter $d_i$
$I$ = electric current
$k$ = Boltzman constant
$k_p$ = flotation rate constant
$n$ = number concentration of bubbles with size distribution rising across the sampling surface
$n_p$ = number concentration of particles
$N$ = local flux along the axis
$N_0$ = flux at $r = r_0$
$\dot{N}_b$ = the number of bubbles generated per unit time
$P$ = atmospheric pressure
$P(d_i)$ = Probability density of bubble of diameter $d_i$
$P(a_i)$ = probability density of bubble of radius $a_i$
$r$ = component in spherical coordinate
$R_g$ = gas constant
$t$ = time
$T$ = absolute temperature
$U_b$ = electrophoretic mobility of bubble
$U_e$ = superficial fluid velocity for bubble of diameter $d_i$
$V_{0i}$ = superficial fluid velocity for bubble of diameter $d_i$
$V_r$ = component of local velocity field $V_0$ in r direction
$V$ = component of local velocity field $V_0$ in $\theta$ direction
$V_s$ = volume of solution in cathode side
$v'$ = the volume of hydrogen bubbles generated per unit time
$X_1$ = the amount of CTAB absorbed by bubbles
$X_2$ = the amount of CTAB absorbed by particles
$-$ = denotes dimensionless value

### Greek Letters

$\alpha$ = coefficient
$\beta$ = correction factor
$\gamma$ = surface tension
$\Gamma$ = surface excess
$\epsilon$ = dielectric constant
$\zeta_B$ = zeta potential of bubble
$\zeta_p$ = zeta potential of particle
$\eta_\epsilon$ = single collection efficiency
$\theta$ = component in spherical coordinate
$\kappa$ = reciprocal Debye length
$\mu$ = fluid viscosity
$\rho$ = fluid density
$\rho_B$ = bubble density
$\sigma$ = geometric standard deviation
$\varphi$ = component in spherical coordinate
$\Phi$ = total interaction potential
$\Phi_A$ = London-Van der Waals potential
$\Phi_R$ = double layer potential
$\psi_i$ = electrostatic potential of surface $i$

## APPENDIX: DECREASE OF CTAB

The surface excess is calculated by the equation as

$$\Gamma = -\frac{C'}{\alpha R_G T}\frac{dr}{dC'} \tag{17}$$

where $C'$ is the bulk concentration of surfactant and $\gamma$ is the surface tension. According to Pethia (1954), $\alpha$ was given a value of 2.

Collins et al. (1978) roughly estimated the "time of equilibrium" as 0.137 s. This result suggests that the bubbles reach equilibrium after moving only a few bubble diameters from the point of formation. When the number of generated bubbles of diameter $d_i$ per unit time is $\dot{N}_b P(d_i)$, the amount of CTAB collected by all the bubbles is calculated as:

$$X_1 = \Gamma \dot{N}_b \sum_i \pi d_i^2 P(d_i) t = \Gamma \dot{N}_b t \pi \sum_i d_i^2 P(d_i) \tag{18}$$

On the other hand, the volume of bubbles generated for $dt$ is expressed as:

$$dV' = \frac{R_G T}{p}\cdot\frac{I}{2Fa}dt = \sum_i \frac{\pi}{6} d_i^3 P(d_i) \dot{N}_b dt \tag{19}$$

Accordingly,

$$\dot{N}_b = \frac{3R_G T I}{\pi P Fa \sum_i d_i^3 P(d_i)} \tag{20}$$

Therefore, substitution of Eq. 20 into Eq. 18 gives:

$$X_1 = \frac{3\Gamma R_G T I \sum d_i^2 P(d_i) t}{PFa \sum_i d_i^3 P(d_i)} = k_0 t \qquad (21)$$

where

$$k_0 = \frac{3\Gamma R_G T I \sum_i d_i^2 P(d_i)}{PFa \sum_i d_i^3 P(d_i)}$$

The amount of CTAB adsorbed by latex particles is also calculated as:

$$X_2 = \Gamma \pi d_p^2 n_p V_s \qquad (22)$$

After all, the time required for the amount of CTAB to decrease by 10% of initial concentration is given as:

$$X_1 + X_2 = 0.1 \times C'V_s \qquad (23)$$

$$t = \frac{0.1C'V_s - X_2}{K_0} \qquad (24)$$

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# Two-Component Laser Doppler Velocimeter Studies of Submerged Jets of Dilute Polymer Solutions

**Axisymmetric submerged jets of dilute solutions of poly(ethylene oxide) and polyacrylamide of high molecular weight were studied using a two-component laser Doppler velocimeter. At from 40–60 jet diameters from the source the small eddies are suppressed in the solutions studied. The behavior of the large eddies depends upon the elastic nature of the solution.**

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## SCOPE

**Submerged jets are an important way to obtain mixing in engineering practice and are also an important experimental arrangement for the study of mixing. For Newtonian fluids the large scale structure of turbulent jets does not depend upon viscosity. Experimental measurements with dilute polymer solutions in submerged jets show that the jet is unaffected by the polymers in some cases (Barker, 1973) and is drastically changed in other cases (Usui and Sano, 1980a). Photographs of jets containing fibers or polymer solutions show large reductions in turbulent mixing and sharp well-defined boundaries between the ambient fluid and the fluid which came out of the jet. These previous studies were imprecise or qualitative but they do suggest that the inviscid character of the jet is sometimes changed.**